Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Turbulence closure for sub grid scale processes in (COSMO and) ICON:

TURBDIFF (<-> TURBTRAN <-> TERRA)

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Some abbreviations and key-words:

ABL: **A**tmospheric **B**oundary **L**ayer

(**G**)**BLA**: (**G**eneralized) **B**oundary **L**ayer **A**pproximation

SAT: **S**urface-to-**A**tmosphere **T**ransfer

SC: **S**ingle **C**olumn

NS: **N**ear **S**urface

SDSS: **S**tandard **D**eviation of local **S**uper-**S**aturation

Related to the **R**oughness **L**ayer:

SAI: **S**urface **A**rea **I**ndex

Special marks for NAMELIST-Parameters:

'parameter' : no special treatment

'parameter' : **optionally perturbed**

"parameter" : **optionally corrected by data-assimilation**

"parameter" : **optionally perturbed and corrected by data-assimilation**

HOC: **H**igher **O**rder **C**losure

SOM: **S**econd **O**rder **M**oment

CDC: **C**onditional **D**omain **C**losure

GS: **G**rid **S**cale

SGS: **S**ub **G**rid **S**cale

LES: **L**arge **E**ddy **S**imulation

Related to **Scale Separation**:

NTC: **N**on-**T**urbulent SGS **C**irculation

STIC: **S**eparated **T**urbulence **I**nteracting with **NTC**s

SSO: SGS **O**rography

TDC: NS **T**hermal-**D**ensity **C**urrent

SHS: **S**eparated **H**orizontal **S**hear

CON: **Con**vection

⟺ **CKE**: **C**irculation **K**inetic **E**nergy

CAT: **C**lear **A**ir **T**urbulence

CIT: **C**onvection **I**nduced **T**urbulence

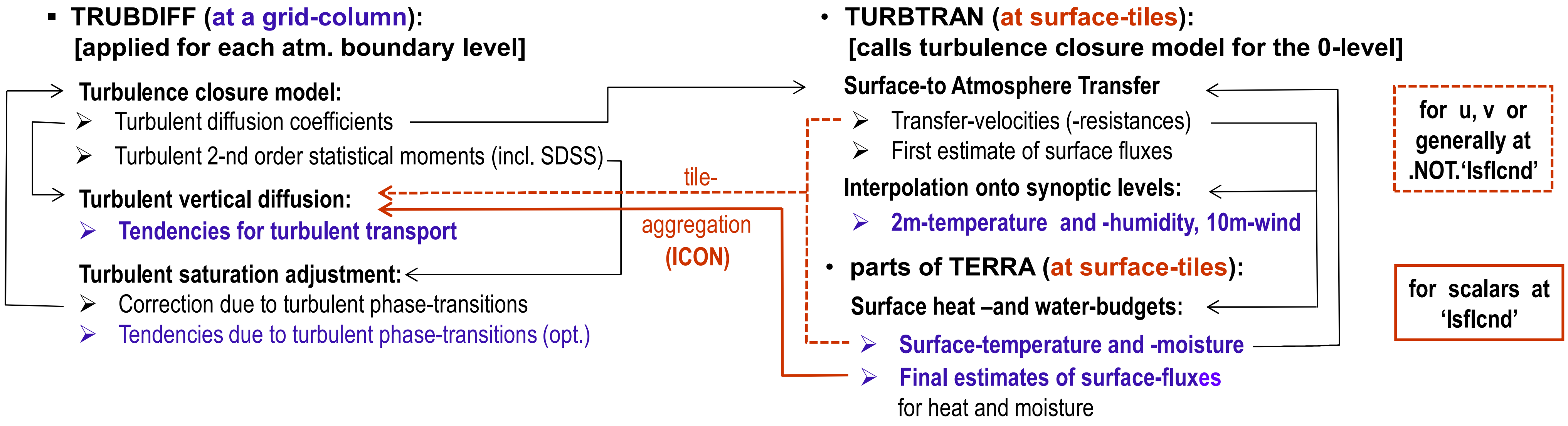
⟺ **TKE**: **T**urbulent **K**inetic **E**nergy

SKE: SG **K**inetic **E**nergy (**CKE** + **TKE**)

General classification of our turbulence scheme and its basic organization :

- **Special solution of the general closure problem**
  - **I. scale-separated trough the constraints of specific closure assumptions**
  - **II. includes 3D-effects and is applicable also for LES**
  - **III. applied also as core of the Surface-to-Atmosphere Transfer (SAT) formulation**

- **Affected ICON-modules** (being **no longer** in common with **COSMO**):



- **TRUBDIFF (at a grid-column):**
  [applied for each atm. boundary level]

Turbulence closure model:
- Turbulent diffusion coefficients
- Turbulent 2-nd order statistical moments (incl. SDSS)

Turbulent vertical diffusion:
- Tendencies for turbulent transport

Turbulent saturation adjustment:
- Correction due to turbulent phase-transitions
- Tendencies due to turbulent phase-transitions (opt.)

tile-

aggregation

(ICON)

- **TURBTRAN (at surface-tiles):**
  [calls turbulence closure model for the 0-level]

Surface-to Atmosphere Transfer
- Transfer-velocities (-resistances)
- First estimate of surface fluxes

Interpolation onto synoptic levels:
- 2m-temperature and -humidity, 10m-wind

- **parts of TERRA (at surface-tiles):**

Surface heat –and water-budgets:
- Surface-temperature and -moisture
- Final estimates of surface-fluxes
  for heat and moisture

for u, v or generally at .NOT.'lsflcnd'

for scalars at 'lsflcnd'

Some general explanations about turbulence reated to NWP :

- **NWP** is based on the numerical integration of a **closed set** of **budget- or state-equations** for all **macroscopic variables** describing the relevant **atmospheric physics**, called the "**first principles**".
- For that purpose, **spatial differentiation operators** needs to be **discretized** on a **numerical grid** of **finite resolution**.
- Due to **sub-grid variability** of physical variables, any **numerical representation** of **spatial differentiation** may be **far from** the **real local value** at the related **grid-points**.
- **Model equations** for **spatially-smoothed variable fields** (called **1-st order statistical moments**) are required, in order to keep this crucial **discretization error** small.
- These **1-st order model equations** can be derived through the application of a **spatial filter** to the **first principles**.
- **Non-linear terms** cause the **closure dilemma**: generation of **new variables** (**higher-order statistical moments**) calling for **additional relations** for **closure** which are **always beyond** the **first principles**!
- **Turbulence** is the **small-scale** part of **sub-gird scale structures** with **special properties** (such as some kind of **isotropy**) used as **closure relations**.
- **Larger sub-grid scale structures** require **other closure relations** being **always additional constraints** that can **never be valid for arbitrary scales**!
- Hence, a full, **multi-scale closure** requires a proper **scale separation**! ➡ **STIC**: **S**eparated **T**urbulence **I**nteracting with **N**on-turbulent **C**irculations

Why do we need parameterizations in a numerical model?

filter operator

molecular flux density

local source term

$$\partial_t\left(\overline{\rho\phi_k}\right)+\nabla\cdot\overline{\left(\underbrace{\boxed{\rho\,\phi_k\,\underline{\mathbf{v}}}+\boxed{\underline{c}^{\phi_k}}}_{\underline{F}^{\phi_k}}\right)}=\boxed{\overline{Q}^{\phi_k}}$$

advection flux-density

- linear or non-linear functions in all model variables $\begin{pmatrix}\rho\\ p\end{pmatrix}, \underline{\phi}$ (including spatial derivatives)
  - dependent on a list of **general valid** parameters $\underline{\alpha}$
  - simplified for efficiency reasons using effective parameters

$\begin{pmatrix}\text{ICON}\\ \text{COSMO}\end{pmatrix}$

$$\phi_k \in \left\{\begin{pmatrix}1\\ p\end{pmatrix}, \underbrace{u, v, w}_{v_1, v_2, v_3}, \overbrace{c_{\binom{v}{p}d}T, q_x}^{\text{scalar variables}}\right\} \qquad p=\rho R_d\cdot\overbrace{\left[1+\left(\frac{R_v}{R_d}-1\right)\cdot q_v-q_c\right]\cdot T}^{T_v}$$

$$Q^{v_i}=-\partial_i p-\delta_{i3}\rho g-2\rho\left(\underline{\mathbf{\Omega}}\times\underline{\mathbf{v}}\right)_i$$

$\phi_k=v_i$: **incompressible Navier-Stokes equation**

local parameterizations:

- molecular flux-densities: $\underline{c}^{\phi_k}=-k^{\phi_k}\nabla\phi_k$
- phase transitions of water (cloud microphysics)
- convergence of radiation flux-densities

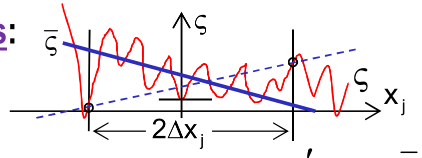
NWP-model = Numerical Simulation of discretized equations in filtered variables:

- Filter may be a moving volume-average dependent on the largest grid-cell dimension $D_g:=\max_j\{\Delta x_j\}$
- Filter removes SGS variability

➢ Approximate Solution of filtered equations

$\Updownarrow$

anti-aliasing:



$\bar{\varsigma}$: filtered (mean) variable and local fluctuation $\varsigma'=\varsigma-\bar{\varsigma}$

(with regard to grid-cell mean)

$\hat{\varsigma}=\dfrac{\overline{\rho\varsigma}}{\bar{\rho}}$: density weighted mean and local fluctuation $\varsigma''=\varsigma-\hat{\varsigma}$

Non-linearity causes generation of statistical moments:

- Non-commutability of filter and (e.g.) multiplication or spatial differentiation

$$\overline{\rho\,\phi_k\,\underline{\mathbf{v}}}=\bar{\rho}\,\hat{\phi}_k\,\underline{\hat{\mathbf{v}}}+\underbrace{\overline{\rho\,\phi_k''\,\underline{\mathbf{v}}''}}_{\text{SGS covariance}} \qquad \overline{\partial_j\varsigma}=\underbrace{\bar{\partial}_j\bar{\varsigma}}_{\text{volume-}}+\underbrace{\overline{\partial_j'\varsigma}}_{\text{surface-}}=\partial_j\bar{\varsigma}+\underbrace{\overline{\partial_j'\varsigma'}}_{\text{full-}}$$

$\overline{\nabla'\cdot\underline{F}^{\phi'}}$: metric correction of flux-divergence

$\overline{\partial_i'p'}$: pressure force by form drag

correction due to embedded R-elements

Equations for GS (filtered) variables are
not closed
due to additional statistical terms!

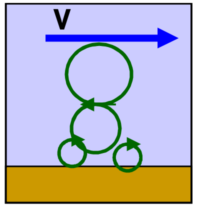
Parameterizations of the additional statistical moments:

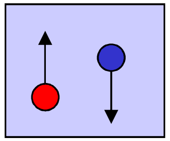
- **Further information** (**assumptions**) about these **additional covariance terms** has to be introduced:

- in terms of functions in all GS model variables (including spatial derivatives) $\begin{pmatrix}\bar{\rho}\\ \bar{p}\end{pmatrix}, \underline{\hat{\phi}}$
  - dependent on a list of related empirical parameters $\underline{\beta}$

GS parameterizations due to *SGS variability*

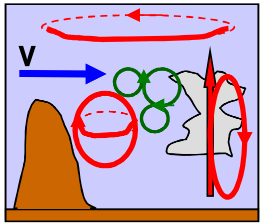
- **Closure assumptions** (including their **parameters**) are **additional constraints** that **can't be general valid**.
  - distinguish **different SGS flow structures** more or less according to the **length scales of their patterns**
  - each with **specific** parameterization assumptions:





STIC

	Turbulence: [TURBDIFF, Raschendorfer] small-scale	**isotropic**; **normal distributed**; only **one** characteristic **turbulent length scale** at each grid point; forced by **shear and buoyancy**
	SGS Circulation: large-scale	**non isotropic**; arbitrarily **skewed** distribution; **coherent** structures of **several independent length scales**; supplied by **various pressure forces**
'ltkecon' <-> dTKEcon	[Tiedke-Bechthod] Convection:	**large vertical scales** of coherence; full microphysics; forced by **buoyancy feed back**
'ltkesso' <-> dTKEsso	[SSO-scheme, Lotts-Miller] Wake eddies:	produced by **blocking** at **SGS surface structures** (form drag forces)
	Breaking gravity wave eddies:	belong to **wave length** of **instable gravity waves** of **arbitrary scales**
'ltketdc' <-> dTKEtdc	*Kata- and anabatic density circulations:*	**direct thermal circulation** forced by lateral **cooling or heating of sloped surfaces** of the earth; dominated by **length scales of SGS surface structures** (like SSO)
'ltkeshs' <-> dTKEshs	[TURBDIFF, Raschendorfer] *Horizontal shear eddies:*	produced by **strong horizontal shear** (e.g. at frontal zones); dominated by **horizontal grid scale >> turbulent length scale**



New general closure strategy according to STIC:

- Describing the crucial **covariance terms** within **different frameworks** all based on **first principals**
- Introducing **closure assumptions** by application of a related **truncation procedure**
- Finding a **flow structure separation** according to the associated **validity of closure assumptions**
- Setting up a **consistently separated set of parameterization schemes**, being (at least potentially) **general valid**
- **Two different closure frameworks** are in use:
  - **Higher order closure (HOC):** Using **budget equations** for needed **statistical moments**, always containing **new ones** (even such of **higher orders -> closure-dilemma**) and **truncating the order of considered moments**.
    - **2nd-order closure:** fits very well to **turbulence**
  - **Conditional domain closure (CDC):** Using **budget equations** for **conditional averages of model variables** (e.g. according to **classes of vertical velocity**) and building the needed covariance terms by an accordingly **truncated statistic, automatically restricting the applicability to associated flow patterns.**
    - **Mass flux closure** (bi- or tri-modal distribution functions): fits very well to **convection**

General second-order budget equation for 2 intensive prognostic variables ϕ and ψ :

particularly applicable for: $\mathrm{SKE} := \frac{1}{2}\sum_{i=1}^{3}\overline{\rho v_i''^2} =: \frac{q^2}{2}$

sub grid scale macroscopic transport

production by **molecular fluxes** from **embedded R-elements**
or by related **tangential** surface stress

$$d_t\left(\overline{\rho\phi''\psi''}\right) := \partial_t\left(\overline{\rho\phi''\psi''}\right) + \bar{\nabla}\cdot\left(\overline{\rho\phi''\psi''}\,\underline{\hat{\mathbf{v}}} + \overline{\rho\phi''\psi''\underline{\mathbf{v}}''} + \overline{\phi''\underline{c}^{\psi}} + \overline{\psi''\underline{c}^{\phi}}\right) = -\overline{\nabla'\left(\phi''\underline{c}^{\psi} + \psi''\underline{c}^{\phi}\right)}$$

shear production

$$-\left(\overline{\underline{c}^{\psi}}\cdot\nabla\hat{\phi} + \overline{\underline{c}^{\phi}}\cdot\nabla\hat{\psi}\right) \quad -\left(\overline{\rho\psi''\underline{\mathbf{v}}''}\cdot\nabla\hat{\phi} + \overline{\rho\phi''\underline{\mathbf{v}}''}\cdot\nabla\hat{\psi}\right)$$

molecular flux density $\underline{c}^{\phi} := -\rho\, k^{\phi}\nabla\phi$ neglected outside the **laminar** layer

$\downarrow$

molecular diffusion coefficient

$$+\left(\overline{\underline{c}^{\psi}\cdot\nabla\phi''} + \overline{\underline{c}^{\phi}\cdot\nabla\psi''}\right)$$

molecular dissipation

$$+\underbrace{\left(\overline{\phi''Q^{\psi}} + \overline{\psi''Q^{\varphi}}\right)}_{-\overline{\phi''\partial_i p}\ ,\quad \psi = v_i}$$

source term correlation

(vanish for **conservative variables**)

$$-\overline{\phi''\partial_i p} = \underbrace{-\bar{\partial}_i\overline{\phi''p'} \qquad -\overline{\partial_i'\phi''p'}}$$

pressure transport $-\bar{\partial}_i\overline{\phi''p'}$

production by **normal** surface stress
of **embedded R-elements**

(e.g., wake-production)

buoyancy source $\delta_{i3}\frac{g}{\hat{\theta}_v}\overline{\rho\phi''\theta_v''} \approx -\overline{\phi''}\,\partial_i\bar{p}$

virtual potential temperature

pressure correlation $+\overline{p'\partial_i\phi''}$

(return-to-isotropy)

red: to be parameterized!

Postulates of pure turbulence:

- **Equilibrium** of the **source terms** in all **reduced 2-nd order budgets**:
  - ○ **traceless elements** $\overline{\rho v_i'' v_j''} - \tfrac{1}{3} q^2$ of the **turbulent stress tensor**

→ - **Neglect** of **local time derivative**

→ - **Neglect** of (**total macroscopic** and **molecular**) **transport**

} at least the sum of both

- **Neglect** of correlations with **pure source terms** of **1-st order budget equations** (except for **momentum**)
- **Neglect** of all **roughness layer terms** (**Internal-BL approximation**) and **molecular shear-production:**
- **Spectral density** of **2nd-order moments** follows a **power law** in terms of **wave length** in **each sample direction** (**inertial sub range spectrum**):

→ - Whole SGS spectrum in a given sampling direction is determined by a **single peak wave length**

- **Peak wave length** is the **same** for samples **in all directions: isotropic length scale** L_p
- **Pressure fluctuations** derivable from: $p + \rho \cdot (\tfrac{1}{2}\underline{v} \cdot \underline{v} + gz) \approx \text{const.}$ within a **L_p-surrounding** according to **Bernoulli's equation**

→ ○ **Pressure correlation** and **dissipation** can be closed -- according to **Rotta** and **Kolmogorov,**
using a **single integral turbulent master length scale** $\ell := \kappa \cdot L;\ L := \min(D_g, L_p);\ \kappa \approx 0.4$ according to **v. Kaman**
  – to be specified **for each location** and **monotonically increasing with height** ------------- according to **Blackadar**
  – including an optional **correction for stable stratification** ----------------------------------- according to **Deardorff** <-> **'a_stab'**

Turbulence is that **class of SGS structures** being in agreement with **these turbulence closure-assumptions!**

The moist extension:

- Inclusion of SGS condensation/evaporation, achieved by:
  - Use of **conservative variables** with respect to condensation: $q_w = q_c + q_v$ (total water), $\theta_w = \theta - \frac{L_c}{c_{p_d}} q_c$ (cloud water potential temperature)

➢ **Correlations** with condensation source terms (including their impact on **buoyancy source-terms**) are considered **implicitly** for **non precipitating clouds**.

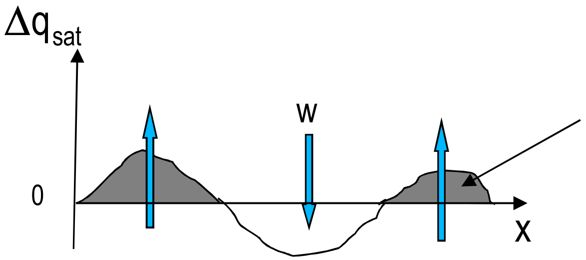
- Solving for:
  - **non-conserved** variables ($\hat{q}_v, \hat{q}_c$ and $\hat{\theta}$)
  - related **statistical moments** (such as $\frac{g}{\hat{\theta}_v} \overline{\rho w'' \theta_v''}$, which is the **buoyancy-source** for **TKE**)
  - **saturated grid-cell fraction** r_c (**cloud-cover**)

by using a **statistical saturation adjustment** --- (according to **Sommeria/Deardorff**):

○ **Normal distribution** of **local super-saturation** Δq_{sat} (of cloud water), which is assumed for **turbulence**, but **not**, .e.g., for **convection**!

○ Expressing **variance** of Δq_{sat} by the three **2-nd order moments** built from θ_w and q_w, being generated by the **turbulence scheme itself**.

<-> 'q_crit' , 'clc_diag'



clouds generated by **sub grid scale condensation** due to turbulent fluctuations of vertical wind speed

Single column solution for turbulent flux densities (based on level 2.5 according M/Y):

1. Applying **closure assumptions** valid for **pure turbulence**:

- ➢ 2-nd order budgets reduce to a **15** X **15** **linear system** of equations

only **traceless** part of the
turbulent **stress tensor**

built of <u>all</u> **second order moments** of the set $\{\ \theta_w,\ q_w,\ \overbrace{u, v, w}\ \}$ of almost **conserved** variables (with respect to evaporation)

θ_w: liquid water potential temperature
q_w: total water mixing ratio

- ➢ and a **prognostic equation for TKE**

2. Using a **Horizontal BL-Approximation (HBLA)** valid for **coarse** (non convection-permitting) resolution **outside the roughness layer**:

- **Neglecting gradients** of **mean quantities** along **horizontal direction** (compared to the vertical direction) as well as **mean vertical wind**.

- ➢ The linear system reduces to **2 linear equations:** for S^M (for momentum) and S^H (for scalars), dependent on

vertical wind shear $F_z^M := (\partial_z \hat{u})^2 + (\partial_z \hat{v})^2$ and **buoyancy (thermal stratification)** $F^H := \frac{g}{\hat{\theta}_v} \cdot \left(\vartheta_w \partial_z \hat{q}_w + r_\theta \partial_z \hat{\theta}_w\right)$

ϑ_w, r_θ: derived thermodynamic parameters dependent on a **turbulent saturation adjustment**

➡ **flux-gradient-representation** of the **turbulent**- **laminar** **flux densities** (where **only** the <u>**vertical**</u> **component** is **relevant** for **strict HBLA**)

$$f_z^\phi := \overline{\rho \phi'' w''} + \overline{c}_z^\phi \approx -\bar{\rho} \cdot \left(K^\phi + k^\phi\right) \cdot \partial_z \hat{\phi}$$

$$K^\phi := \ell\ S^\phi \cdot q$$

turbulent diffusion coefficient ,

ℓ: turbulent master length scale
S^ϕ: stability function
q: turbulent velocity scale $\leftarrow \left(\frac{1}{\bar{\rho}} \underbrace{\sum_{i=1}^{3} \overline{\rho v_i''^2}}_{2 \cdot \mathrm{TKE}}\right)^{1/2}$

artificially <u>limited</u> by specific
minimal diffusion coefficients
substituting the impact of
missing STIC-terms!

STIC: Coarse resolution extension via formal scale-separation:

- **Basic assumption**: **Turbulence approximations** can be assigned to **all horizontal scales not larger than the sub-grid turbulent peak wave-lenght** **L** (mainly dependent on the **distance from the surface of the earth)**

- **Method:** **Spectral separation** by

i.	considering **budgets** with respect to the **separation scale** $L \le D_g$	double averaging
ii.	**averaging** these budgets along the **whole control volume**	

➢ **1-st order budgets:** **SGS contributions** by **turbulence and NTCs**

$$\overline{\rho\phi\psi} = \bar{\rho}\hat{\phi}\hat{\psi} + \overline{\overline{\rho\phi''\psi''}\Big|_L} + \overline{\bar{\rho}\Big|_L \hat{\phi}\Big|_L^{\,''} \hat{\psi}\Big|_L^{\,''}}$$

: with respect to the separation scale L

➢ **2-nd order budget** for **scale-separated turbulent moments** contain **novel scale-interaction terms**

- due to **non-linearity** of **shear-terms**

$$D_t\left(\overline{\overline{\rho\phi''\psi''}\Big|_L}\right) = \cdots \left[-\overline{\overline{\rho\phi''\underline{\mathbf{v}}''}\Big|_L \cdot \left(\nabla\hat{\psi}\right)\Big|_L} + -\overline{\overline{\rho\psi''\underline{\mathbf{v}}''}\Big|_L \cdot \left(\nabla\hat{\phi}\right)\Big|_L} \right] + \cdots$$

$$\overline{\overline{\rho\phi''\underline{\mathbf{v}}''}\Big|_L} \approx -\bar{\rho}\cdot K^{\phi}\cdot\nabla\hat{\phi} \qquad \underbrace{-\rho\ \overline{\overline{\phi''\underline{\mathbf{v}}''}\Big|_L}\cdot\nabla\hat{\psi}} \quad \underbrace{-\rho\ \overline{\overline{\phi''\underline{\mathbf{v}}''}\Big|_L^{\,'}\cdot\left(\nabla\hat{\psi}\right)\Big|_L^{\,'}} \quad -\overline{\overline{\rho\psi''\underline{\mathbf{v}}''}\Big|_L^{\,'}\cdot\left(\nabla\hat{\phi}\right)\Big|_L^{\,'}}} \quad \underbrace{-\overline{\overline{\rho\psi''\underline{\mathbf{v}}''}\Big|_L}\cdot\nabla\hat{\phi}} \qquad \overline{\overline{\rho\psi''\underline{\mathbf{v}}''}\Big|_L} \approx -\bar{\rho}\cdot K^{\psi}\cdot\nabla\hat{\psi}$$

turbulent shear term **NTC shear production** $\Delta Q_C^{\phi\psi}$ **turbulent shear term**

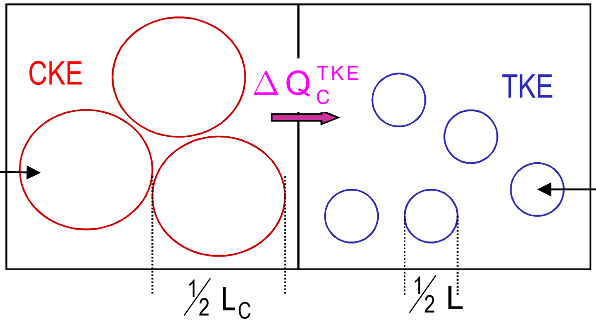
scale-interaction term

STIC: Scale transfer of Sub-grid Kinetic Energy (SKE):

production terms dependent on:

specific length scales L_C
and a **specific velocity**
scale

$$q_c := \left(\frac{1}{\bar{\rho}}\underbrace{\overline{\bar{\rho}|_L\,\hat{\underline{v}}|_L''\cdot\hat{\underline{v}}|_L''}}_{2\cdot CKE}\right)^{1/2}$$



production terms depend on:

turbulent length scale L and
the **turbulent velocity scale**

$$q := \left(\frac{1}{\bar{\rho}}\underbrace{\overline{\overline{\rho\underline{v}''\cdot\underline{v}''}|_L}}_{2\cdot TKE}\right)^{1/2}$$

The **scale interaction term** shifts **SKE** (or any other variance) form the **NTC part of the spectrum (CKE)** towards the **turbulent part (TKE)** by virtue of **shear** generated by **circulation flow patterns**.

$$\Delta Q_M^{SKE} := -\sum_{i=1}^{3}\overline{\rho v_i''\underline{v}''}\cdot\nabla\hat{v}_i$$

$$\Delta Q_M^{CKE} := -\sum_{i=1}^{3}\overline{\bar{\rho}|_L\,\hat{v}_i|_L''\,\hat{\underline{v}}|_L''}\cdot\nabla\hat{v}_i$$

$$\Delta Q_M^{TKE} =: -\sum_{i=1}^{3}\overline{\overline{\rho v_i''\underline{v}''}|_L}\cdot\nabla\hat{v}_i$$

$$\Delta Q_C^{TKE} := -\sum_{i=1}^{3}\overline{\overline{\rho v_i''\underline{v}''}|_L'\cdot\left(\nabla\hat{v}_i\right)_{|L}'}$$

- **Closure of STIC-terms:**

according to any
specific parameterization

so far treated **similarly** to
turbulent dissipation
(aimed to be modified)

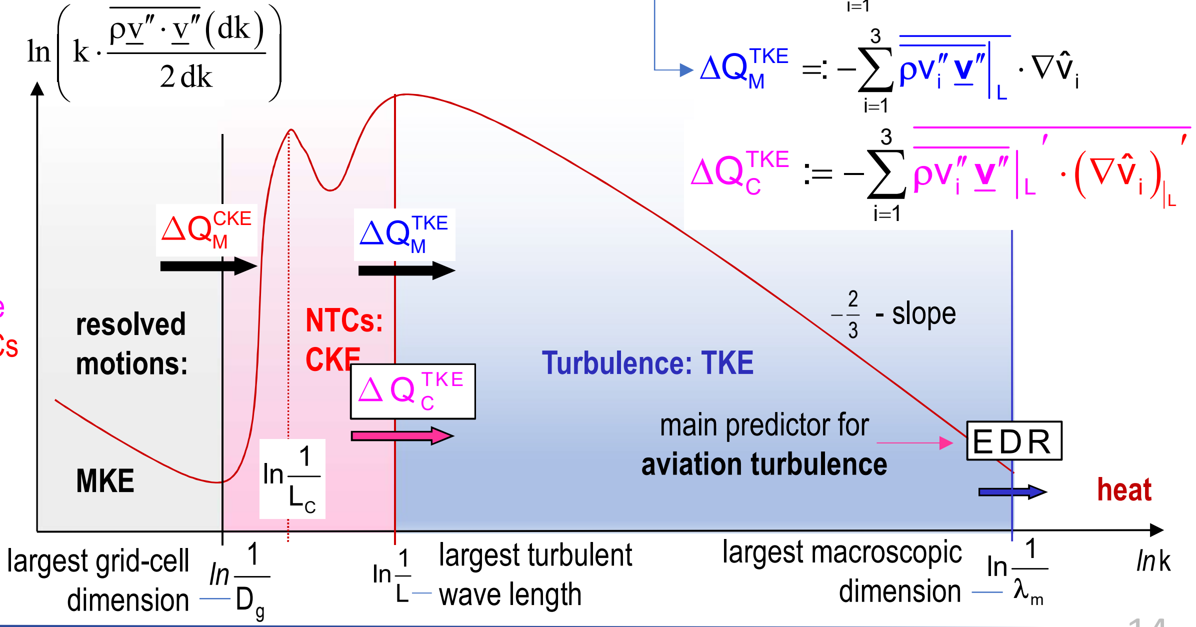
$$\textbf{Production\_of\_CKE}\,(L_C, q_C) = \Delta Q_C^{TKE}(L_C, q_C)$$

$$=: K^M \Delta F_C^M$$

sub-grid scale
shear by NTCs

turbulent diffusion coefficient
for momentum

if “**dependent** on (L_C, q_C)“ then
  specific new parameterization:
  dTKEshs, dTKEdtc
else
  derived from existing NTC-schemes:
  dTKE_sso, dTKE_con
end If



STIC: Closure by means of Gerneralized tangent-to-stream BL-Approximation (GBLA):

Tilted, local and temporal Tangent-to-Stream (TS) system

- **n-coordinate points in direction of strongest wind-speed ascend**

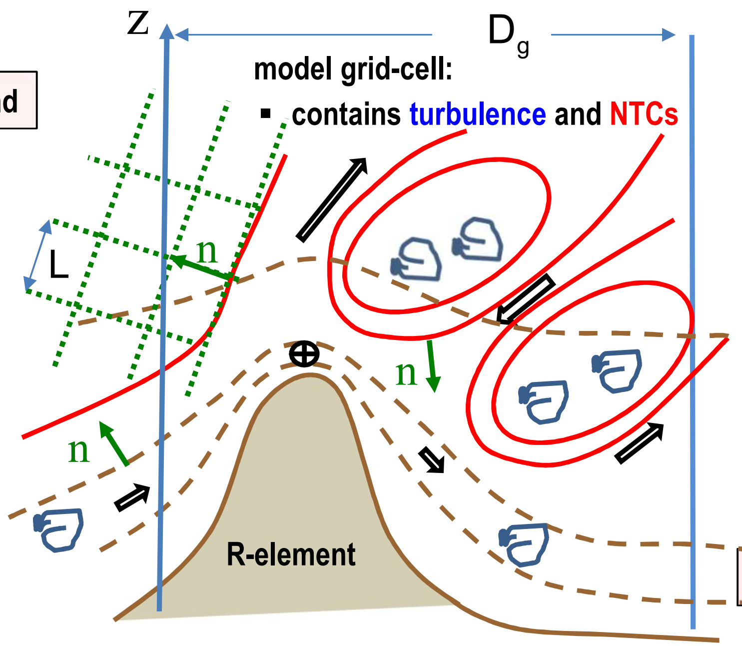
- **virtual separation grid of scale L resolves all NTCs (only turbulence is sub-grid)**

'a_stab'

$$\frac{\kappa}{\ell} = \frac{1}{L} = \frac{1}{\delta} + \frac{1}{\min\left\{D_g, L_p^{max}\right\}} + \kappa \cdot a_{stab} \frac{\sqrt{F^H_{\geq 0}}}{q}$$

maximal-possible turbulent wave-length: 'tur_len'

separation scale = sub-grid turbulent peak wave-length



model grid-cell:

- **contains turbulence and NTCs**

Tangent-to-Stream (TS) surface (related to L-filtered wind-field):

- **negligible tangential gradients of L-filtered variables**
- **may be multi- and even dis-connected**
- **may intersect δ-surfaces**
- **may coincide with δ-surfaces**

δ - surface:

- **constant turbulent distance**
- **δ is also a vertical coordinate**

Tangent-to-Stream Approximation (TSA):

- **SC turbulence closure** can **approximately** be applied to a **tilted, local and temporal L -TS system (tilted SC-solution):**
  - **Gravity no longer aligned with normal direction: buoyancy** remains being dependent on a **vertical gradient:**
    $$F^H = \frac{g}{\hat{\theta}_v} \cdot \left(\vartheta_w \partial_z \hat{q}_w + r_\theta \partial_z \hat{\theta}_w \right)$$
  - **Vertical wind-shear => Normal wind-shear with respect to L-TS system:**
    $$\overline{\left(\partial_n \hat{u}\right)^2_L} + \overline{\left(\partial_n \hat{v}\right)^2_L} + \overline{\left(\partial_n \hat{w}\right)^2_L} = F^M = \left(\partial_z \hat{u}\right)^2 + \left(\partial_z \hat{v}\right)^2 + F^M_h + \Delta F^M_c$$
    STIC-term

➡ **Extended 3D flux-gradient-representation of the turbulent flux-components** based on the **double filter:**

$$\overline{\overline{\rho \phi'' v''_j}}\Big|_L \approx -\bar{\rho} K^\phi \partial_j \hat{\phi} \quad \text{(for scalars)}$$

$$\overline{\overline{\rho v''_i v''_j}}\Big|_L \approx \bar{\rho} \frac{q^2}{3} \delta_{ij} - \bar{\rho} K^{v_i} \left[\partial_i \hat{v}_j + \partial_j \hat{v}_i \right]$$

$$\nabla \cdot \overline{\overline{\rho v''_i \underline{v}''}}\Big|_L \overset{\overbrace{\nabla\cdot\hat{\underline{v}}\approx 0}}{\approx} \partial_i \left(\bar{\rho} \frac{q^2}{3} \right) + \nabla \cdot \left(-\bar{\rho} K^{v_i} \nabla \hat{v}_i \right)$$

Separated semi-parameterized TKE-equation with Scale-Interaction sources:

$e_{ks} := \tfrac{1}{2}\underline{v}'' \cdot \underline{v}''$

$\mathrm{TKE} := \overline{\overline{\rho e_{ks}}\big|_L}$ ← double-filter with respect to separation scale L

grid-scale vertical — grid-scale horizontal — sub-grid scale 3D — shear

$$\overset{!!}{\overbrace{\partial_t\left(\tfrac{1}{2}\bar{\rho}q^2\right)}} = \nabla\cdot\left(\begin{array}{l}\tfrac{1}{2}\bar{\rho}q^2\,\hat{\underline{v}} \\ +\overline{\overline{\rho e_{ks}}\big|_L{}'\,\hat{\underline{v}}'\big|_L} \\ +\overline{\overline{\left(\rho e_{ks}+p'\right)\underline{v}''}\big|_L}\end{array}\right) + \frac{g}{\hat{\theta}_v}\overline{\overline{\rho\theta_v''w''}\big|_L} + \overbrace{\left[-\sum_{i=1}^{3}\overline{\overline{\rho v_i''\underline{v}''}\big|_L}\cdot\nabla\hat{v}_i\right] + \left[-\sum_{i=1}^{3}\overline{\overline{\rho v_i''\underline{v}''}\big|_L{}'\cdot\left(\nabla\hat{v}_i\right)\big|_L{}'}\right]}^{K^M\cdot\left(F_z^M + F_h^M + \Delta F_C^M\right) = K^M F^M \geq 0} + \left[-\bar{\rho}\frac{q^3}{\alpha^{MM}\ell}\right]$$

time tendency

transport:
(advection
+ circ. diffusion
+ turb. diffusion)

buoyancy production

labil: > 0
neutral: $= 0$
stabil: < 0

shear production by the mean flow

$\boxed{\geq 0}$

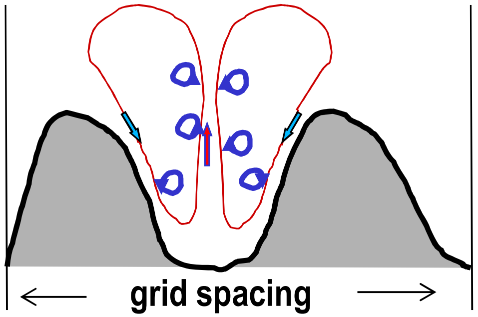
shear production by **NTCs**

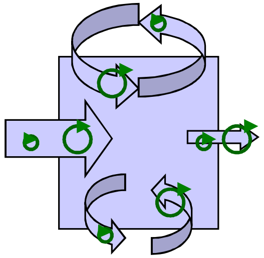
$\boxed{\geq 0}$

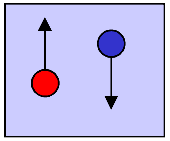
eddy-dissipation rate (**EDR**)

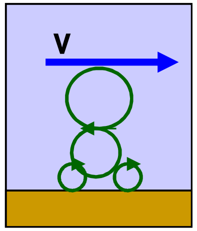
$\boxed{< 0}$

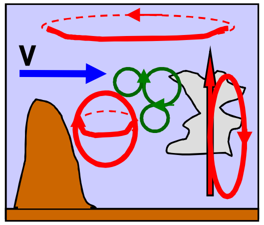
Shear-production of TKE by katabatic TDC:

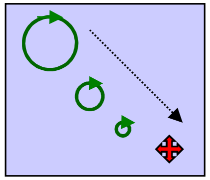












- ➢ **Turbulence closure remains simple (restriction to isotropic scales and normal-distributed fluctuations) !**
- ➢ **Additional shear by NTCs** generates **more physically based turbulent mixing, particularly for stable stratification!**

Iterative solution for TKE, the stability-functions and cloud-fraction:

$$\alpha := \partial_T q_{vs}\left(\hat{T}\right) \qquad T_c := \frac{L_c}{c_{p_d}} \qquad \vartheta_c := \frac{T_c}{r_p} \qquad \vartheta_v := r_T \cdot \left(r_v \vartheta_c - \frac{R_v}{R_d}\hat{T} \right)$$

$$r_\theta := r_v - \mathbf{r_c}\alpha\vartheta_v \qquad \vartheta_w := \left(\frac{R_v}{R_d} - 1 \right) \cdot \hat{\theta} + \frac{\mathbf{r_c}}{r_p} \cdot \vartheta_v$$

cloud-fraction $\mathbf{r_c}$

$$r_T := \frac{1}{1+\alpha T_c}$$

$$r_v := 1 + \left(\frac{R_v}{R_d} - 1 \right) \cdot \hat{q}_v - \hat{q}_c$$

$$r_p := \left(\frac{\bar{p}}{p_r} \right)^{\frac{R_d}{c_{p_d}}}$$

due to GS horizontal shear

$$\mathbf{F^M} := \underbrace{\left(\bar{\partial}_z \hat{u}\right)^2 + \left(\bar{\partial}_z \hat{v}\right)^2}_{F_z^M} + \mathbf{F_h^M} + \mathbf{\Delta F_C^M} \; ; \quad F^H := \frac{g}{\hat{\theta}_v} \cdot \left(r_\theta \cdot \overbrace{\bar{\partial}_z \hat{\theta}_w + \bar{\partial}_z \hat{q}_w}^{S^H \uparrow \ell} \cdot \vartheta_w \right);$$

$$\frac{1}{\ell(z)} = \frac{1}{\kappa L} \approx \frac{1}{\kappa z} + \underbrace{\frac{1}{\ell_m}}_{\kappa \cdot \text{'tur\_len'}} + \overset{\text{'a\_stab'}}{a_{stab}^l} \frac{\sqrt{F^H}}{q}$$

$$\boxed{\partial_t \left(\frac{1}{2}\bar{\rho} q^2 \right) + \bar{\partial}_z \left[-\bar{\rho}\,\ell\, S^q\, q\, \bar{\partial}_z \left(\frac{1}{2} q^2 \right) \right] = \bar{\rho} q \ell \cdot \left[S^M \mathbf{F^M} - S^H F^H \right] - \frac{q^3}{\alpha^{MM}\ell}} \longrightarrow \mathbf{G^M} := \frac{\ell^2}{q^2} \cdot \mathbf{F^M} \geq 0 \quad G^H := \frac{\ell^2}{q^2} \cdot F^H$$

$$\overbrace{\left[\frac{1}{\alpha^H} + \left(3\alpha^{HH} + 12\alpha^M \right) \cdot G^H \right]}^{=:a_{HH}} \cdot S^H + \overbrace{6\alpha^M \mathbf{G^M}}^{=:a_{HM}} \cdot S^M = 1 - 3c^H =: b_H$$

$$\underbrace{\left[\left(9\alpha^H + 12\alpha^M \right) \cdot G^H \right]}_{=:a_{MH}} \cdot S^H + \underbrace{\left[\frac{1}{\alpha^M} + 9\alpha^H G^H + 6\alpha^M \mathbf{G^M} \right]}_{=:a_{MM}} \cdot S^M = 1 - 3c^M =: b_M$$

$\alpha^M = 0.92, \quad \alpha^{MM} = 16.6, \quad c^M = 0.08$

$\alpha^H = 0.74, \quad \alpha^{HH} = 10.1 \quad c^H = 0.0$

$$S^M = \frac{b_M a_{HH} - b_H a_{MH}}{a_{HH} a_{MM} - a_{HM}\, a_{MH}} \qquad S^q = 0.20 \qquad S^H = \frac{b_H a_{MM} - b_M a_{HM}}{a_{HH} a_{MM} - a_{HM}\, a_{MH}}$$

Matthias Raschendorfer

Main characteristics of the turbulence scheme TURBDIFF:

- Based on 2^{nd} order closure on level 2.5 according to M/Y: contains a **prognostic TKE-equation**

<-> **‘imode_turb’**
tran

- Includes SGS saturation adjustment: it is a **moist turbulence scheme**
  using **conservative variables** with respect to
  condensation/evaporation

<-> **‘icldm_turb’**
tran
‘itype_wcld’
‘q_crit’

- Applies a (generalized) **BLA**: mainly a **SC-scheme**
  but optionally extendable to **horizontal shear contributions**

<-> **‘itype_sher’**
‘ltkeshs’

- Conceptually separated from description of other SGS structures: it’s a **STIC-scheme**
  with additional
  scale-interaction terms

- Applicable also within the roughness layer: it is the core of the **SAT-scheme**

New Development

<-> **‘pat_len’**
‘ltkesso’
shs
con

- ➢ Provides: ∂_t **(1st-order progn. model variables)**$|_{\text{SGS turbulent procs.}}$ ≈ turbulent **[vertical-diffusion**
  + (phase transitions)**]**

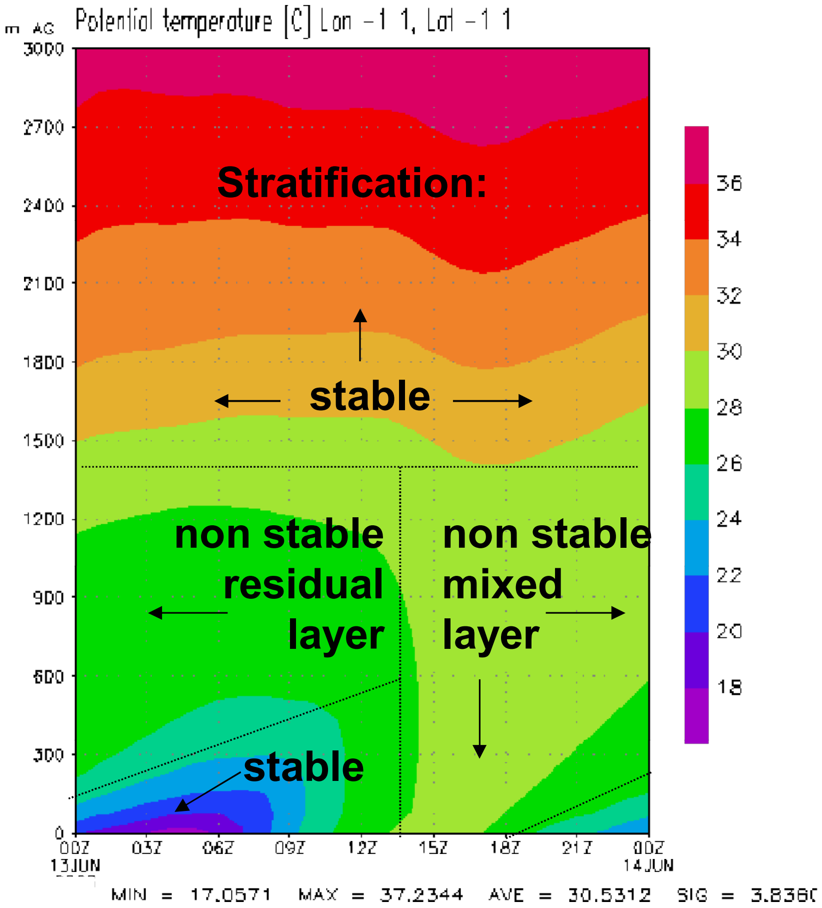
<-> **‘lsflcnd’**
‘lexpcor’
‘impl_s’
t

- ➢ Main contributor to **diurnal variations** within the ABL:
  - **daytime heating and mixing; nocturnal cooling;**

  SGS cloudiness

<-> **‘clc_diag’**
‘q_crit’

Time-Height cross sections of dynamic variables from a COSMO-simulation:



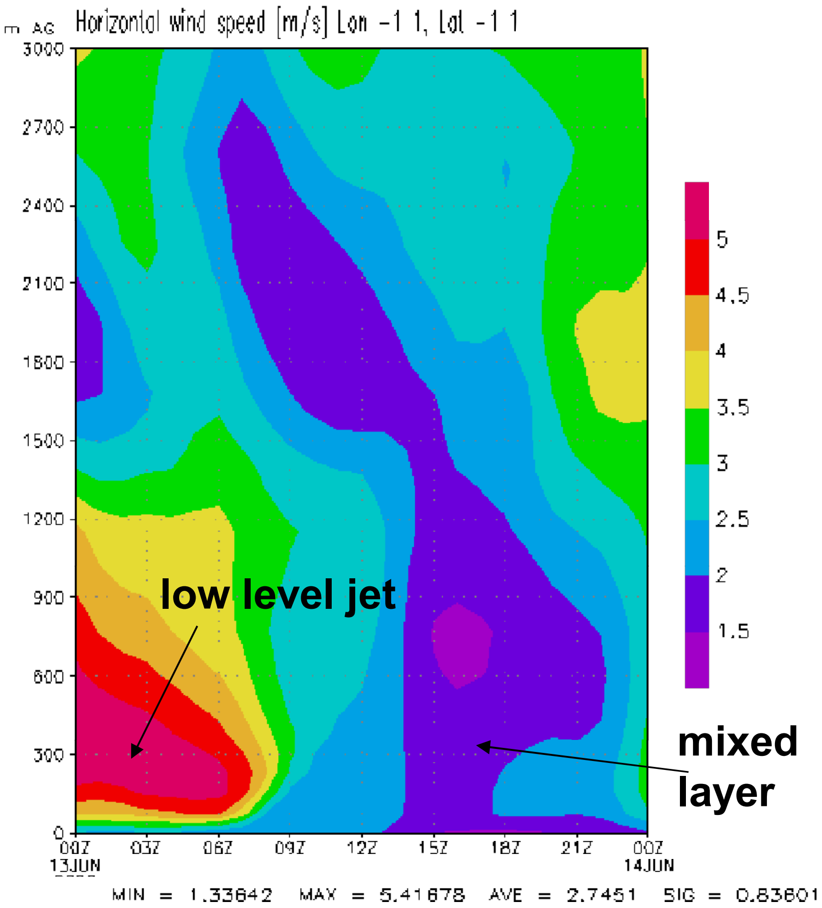
Stratification:

stable

non stable residual layer

non stable mixed layer

stable

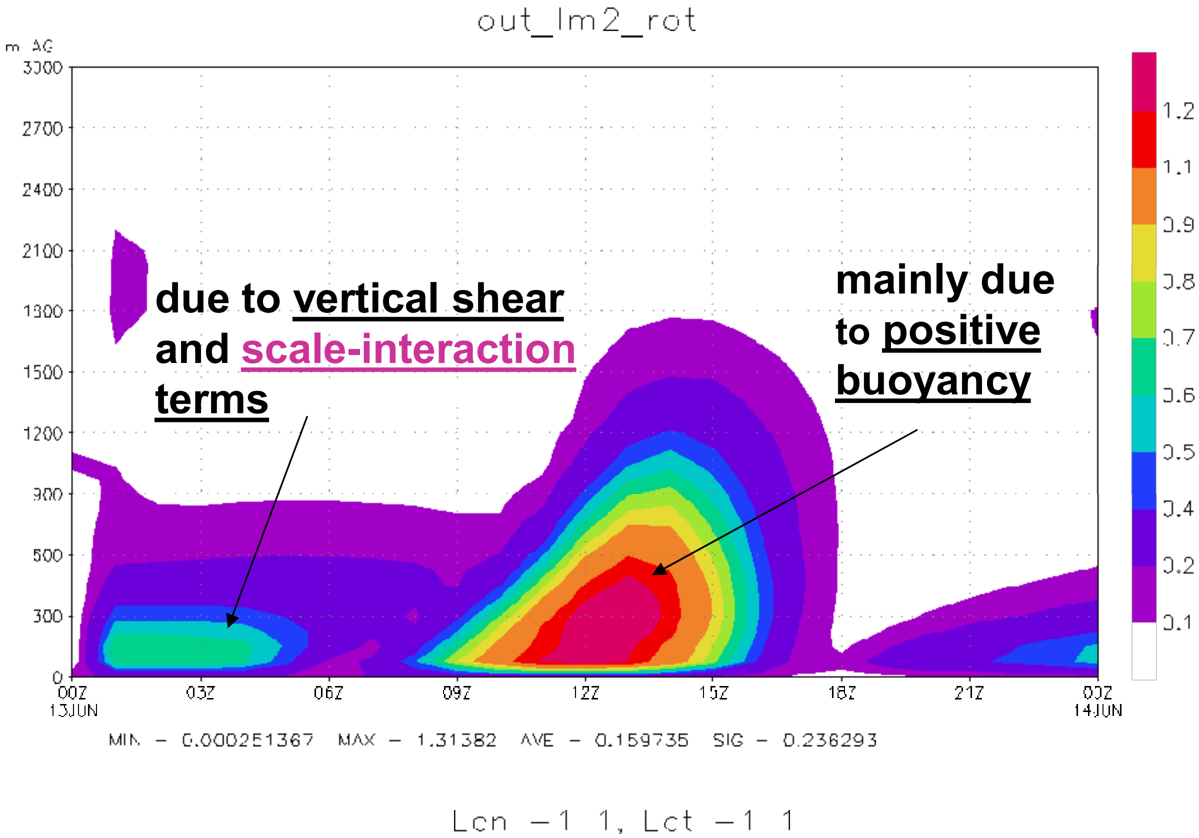


low level jet

mixed layer

Time-Height cross section of TKE from a COSMO simulation:

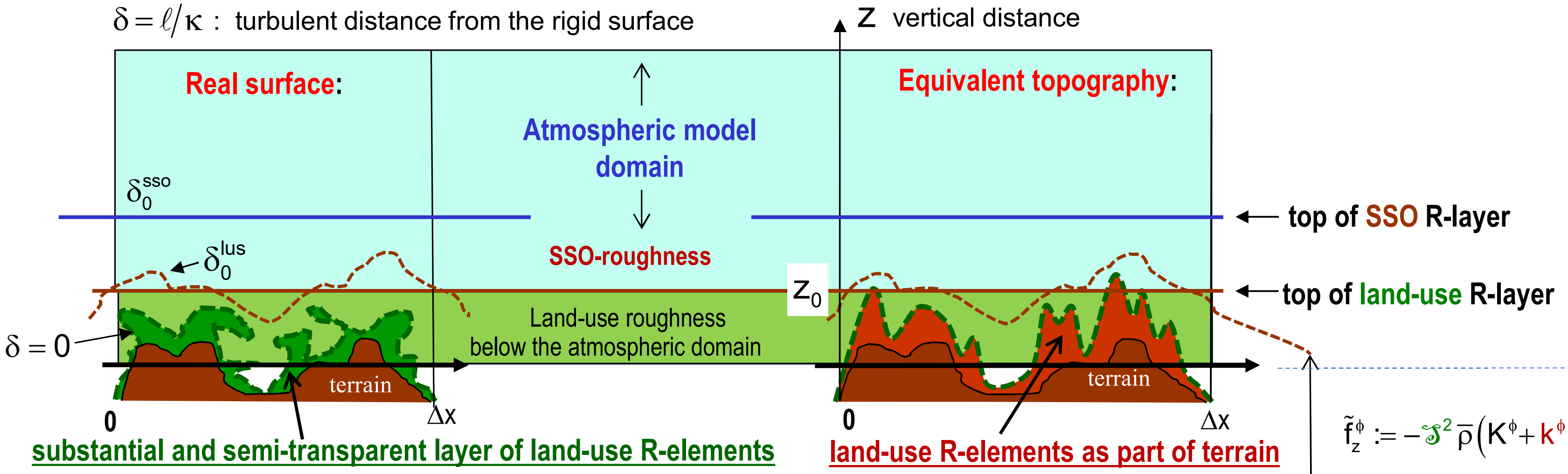
Mass-density of Turbulent Kinetic Energy (TKE) [m^2/s^2]



due to **vertical shear** and **scale-interaction terms**

mainly due to **positive buoyancy**

Treatment of the roughness layer : TUBDIFF <-> TURBTRAN <-> TERRA:



vertically resolved:
drag-impact by SSO-scheme
no metric flux-div.-impact

vertically not resolved:
part of **transfer-resistance**

$$\tilde{f}_z^\phi := -\Im^2 \overline{\rho}\left(K^\phi + k^\phi\right)\partial_z \hat{\phi}$$

effective vertical turbulent-molecular flux

- At the **top** of the **land-use R-layer**, it holds:

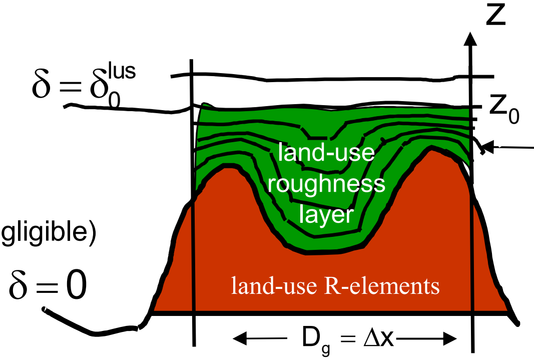
  $z = \delta_0^{lus} =: z_0$

- At the **top** of the **land-use R-layer** **and above**, it holds:

  von-Kaman constant

  $\ell \approx \kappa \cdot z \quad k^\phi \ll K^\phi$ (laminar diffusion negligible)

  $(\partial_\delta z =:) \Im \approx 1 \approx S_{ai}^{sso}$ **(HBLA)**



- δ-surfaces, being **normal** to **local** turbulent and molecular scalar flux densities.
- They can be constructed by a **shifted equiv. topography** without the **small scale modes** that do not contribute to roughness terms in the respective level (for any variable ϕ).
- Their magnitude is $\partial_\delta z = \Im$ times that of the horizontal plane

Surface-Area Index (SAI)

- z_0 is the **roughness length** and belongs to the **lowest level** with $\ell \approx \kappa \cdot z$

Transfer-layer resistance-chain and its dependency on turbulence:

$$r_{SA}^{\phi} := \int_{\delta_S=0}^{\delta_A} \frac{d\delta}{\mathfrak{S}^2 \left(K^{\phi} + k^{\phi}\right)} = r_{S0}^{\phi} + r_{0A}^{\phi}$$

transfer-layer transport resistance

$$\varphi_{XY}^{\phi} := \kappa u_0^{\phi} \cdot r_{XY}^{\phi}$$

dimensionless partial transport resistance

$$\varphi_{0A}^{\phi} = \begin{cases} \dfrac{1}{1-\gamma_s^{\phi}} \cdot \ln\left[\dfrac{z_A}{z_0 + \gamma_s^{\phi} \cdot h_A}\right] & \text{unstable}, s = +1 \\ \left(1-\gamma_s^{\phi}\right) \cdot \ln\left(\dfrac{z_A}{z_0}\right) + \gamma_s^{\phi} \dfrac{h_A}{z_0} & \text{stable}, s = -1 \end{cases}$$

$$h_X := z_X - z_0$$

atmospheric height

$$\gamma_s^{\phi} := \frac{z_0}{h_P}\left[\left(\frac{u_P^{\phi}}{u_0^{\phi}}\right)^{s} - 1\right]$$

stratification parameter

$$\varphi_{S0}^{M} = 0$$

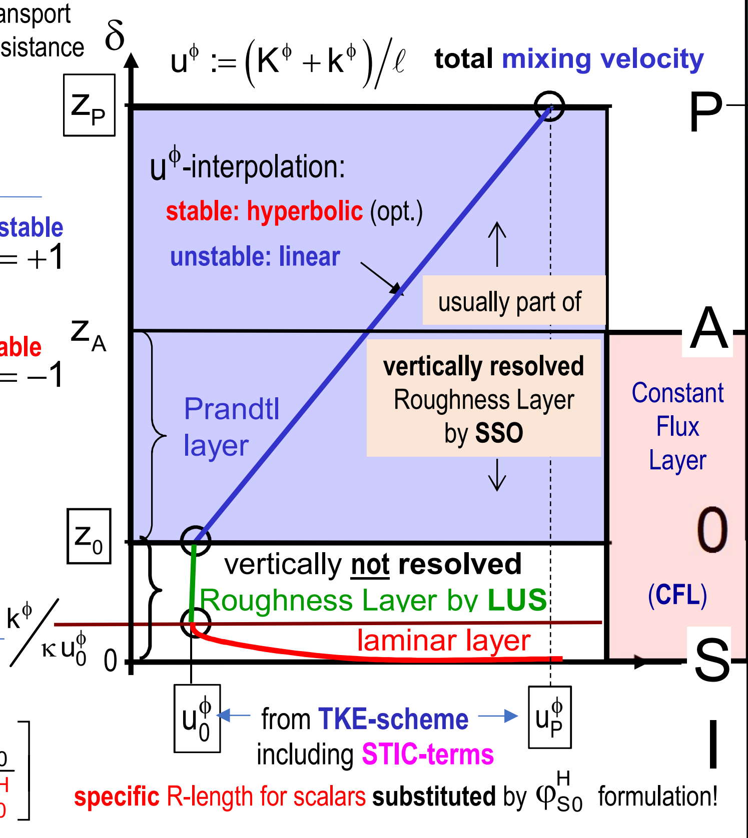
$$\varphi_{S0}^{H} = \underbrace{\frac{1}{z_0}\int_0^{z_0}\frac{d\delta}{\mathfrak{S}}}\cdot\left(\boxed{\lambda^{H}} + \ln\frac{K_0^{H}}{k^{H}}\right) =: \ln\left[\frac{z_0}{z_0^{H}}\right]$$

land-point: S_{ai}^{lus} = ‘c_lnd’ + ’plcov’· ’lai’

sea-point: ‘c_sea’

$$\lambda^{H} = \underline{\text{“rlam\_heat”}} \cdot \begin{cases} 1 & \text{not for vapor (e.g. heat)} \\ \underline{\text{‘rat\_lam’}} & \text{only for vapor} \end{cases}$$

$$\underline{\text{“rlam\_heat”}} \cdot \underline{\text{’rat\_sea’}}$$



δ

$u^{\phi} := \left(K^{\phi} + k^{\phi}\right)/\ell$ **total mixing velocity**

z_P P

u^{ϕ}-interpolation:

stable: hyperbolic (opt.)

unstable: linear

usually part of

vertically resolved Roughness Layer by **SSO**

z_A A

Prandtl layer

Constant Flux Layer

0

(CFL)

z_0

vertically **not resolved** Roughness Layer by **LUS**

laminar layer

$k^{\phi}/\kappa u_0^{\phi}$ 0

S

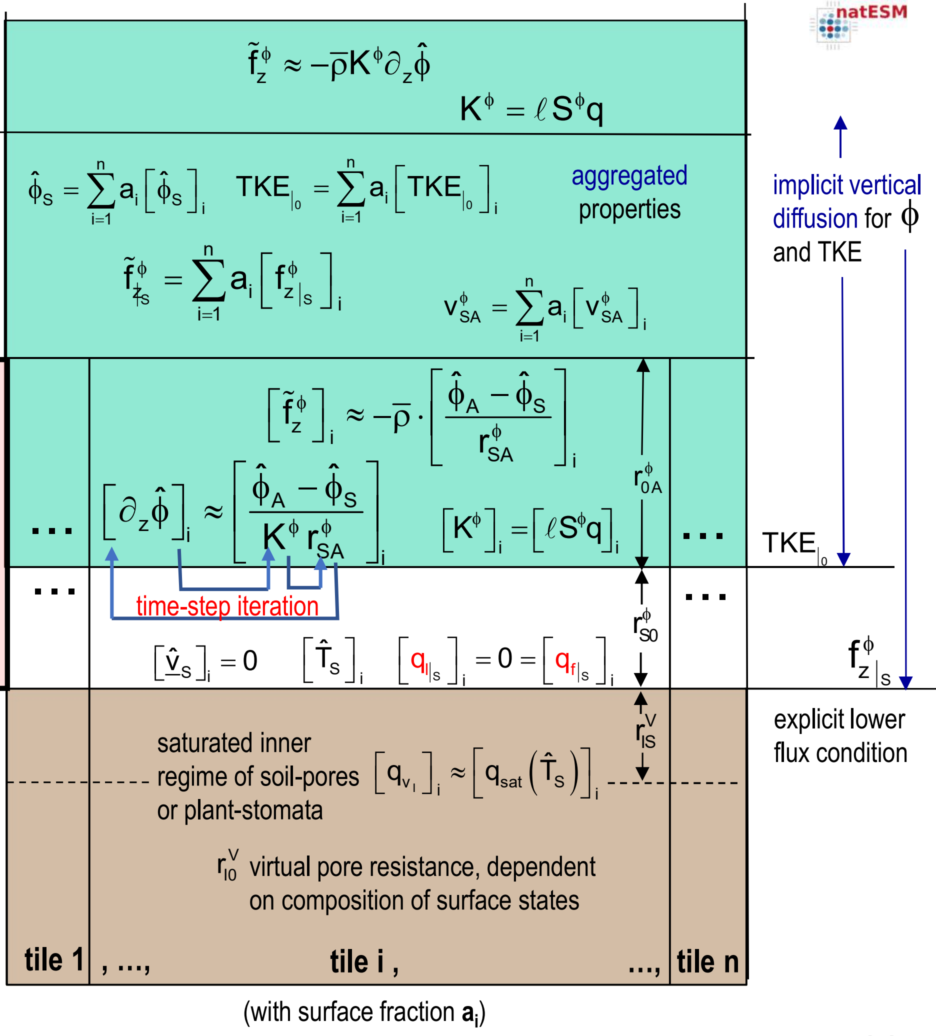
I

u_0^{ϕ} ← from **TKE-scheme** → u_P^{ϕ}

including **STIC-terms**

specific R-length for scalars **substituted** by φ_{S0}^{H} formulation!

$\phi := \phi_k$ **any single prognostic variable**



$$\tilde{f}_z^{\phi} \approx -\bar{\rho} K^{\phi} \partial_z \hat{\phi}$$

$$K^{\phi} = \ell S^{\phi} q$$

$$\hat{\phi}_S = \sum_{i=1}^{n} a_i \left[\hat{\phi}_S\right]_i \qquad TKE_{|_0} = \sum_{i=1}^{n} a_i \left[TKE_{|_0}\right]_i$$

aggregated properties

$$\tilde{f}_{z_S}^{\phi} = \sum_{i=1}^{n} a_i \left[f_{z|_S}^{\phi}\right]_i \qquad v_{SA}^{\phi} = \sum_{i=1}^{n} a_i \left[v_{SA}^{\phi}\right]_i$$

implicit vertical diffusion for ϕ and TKE

$$\left[\tilde{f}_z^{\phi}\right]_i \approx -\bar{\rho} \cdot \left[\frac{\hat{\phi}_A - \hat{\phi}_S}{r_{SA}^{\phi}}\right]_i$$

$$\left[\partial_z \hat{\phi}\right]_i \approx \left[\frac{\hat{\phi}_A - \hat{\phi}_S}{K^{\phi}\, r_{SA}^{\phi}}\right]_i \qquad \left[K^{\phi}\right]_i = \left[\ell S^{\phi} q\right]_i$$

r_{0A}^{ϕ}

… …

$TKE_{|_0}$

time-step iteration

r_{S0}^{ϕ}

$$\left[\underline{\hat{v}}_S\right]_i = 0 \qquad \left[\hat{T}_S\right]_i \qquad \left[q_{l|_S}\right]_i = 0 = \left[q_{f|_S}\right]_i$$

$f_{z|_S}^{\phi}$

r_{lS}^{V}

explicit lower flux condition

saturated inner regime of soil-pores or plant-stomata $\left[q_{v_l}\right]_i \approx \left[q_{sat}\left(\hat{T}_S\right)\right]_i$

r_{l0}^{V} virtual pore resistance, dependent on composition of surface states

tile 1 , …, **tile i ,** …, **tile n**

(with surface fraction $\mathbf{a_i}$)

Calculation of surface-concentrations and interpolation onto synoptic levels:

- Non-slip condition for momentum: $u_S = 0 = v_S$
- Analyzed SST for temperature and saturation humidity at the sea surface
- Surface-heat-budget for temperature at snow-fee (Sf) and snow-covered (Sn) surface:

  $\hat T_{Sf}$ and $\hat T_{Sn}$ calculated in multi-layer soil module TERRA

- Consideration of surface-specific pore resistances $r^{q_v}_{l_x S_x}$ for specific humidity:

$$\frac{\hat q_A - \hat q_S}{r^H_{SA}} = -\frac{f^{q_v}_S}{\bar\rho_S} = \sum_x a_{S_x} \frac{\hat q_S - q_{sat}\left(\hat T_{S_x}\right)}{r^{q_v}_{l_x S_x}}$$

water vapor flux-density
at the lower boundary

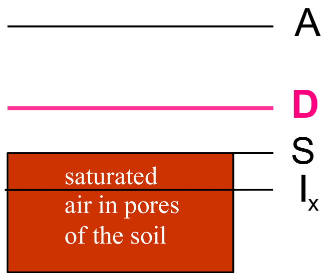
relative area weight
(of the surface-class)

$$a_{S_x} = \frac{1}{S^{lus}_{ai}} \cdot \begin{cases} \text{'}\underline{\text{c\_soil}}\text{'} & \text{evaporating bare-soil surface} \\ \text{'lai'} \cdot \text{'plcov'} & \text{transpiring vegetation surface} \\ \text{'c\_lnd' - '}\underline{\text{c\_soil}}\text{'} & \text{sealed surface} \end{cases}$$

- ○ at water, ice and snow: $r^{q_v}_{l_x S_x} = 0$ => $\hat q_{vS} = q_{sat}\left(\hat T_{S_x}\right)$ (potential evaporation)
- ○ at vegetation/bare-soil: $r^{q_v}_{l_x S_x}$ is the stomata/pore-resistance (restricted evapo-transpiration)
- ○ at sealed surfaces: $r^{q_v}_{l_x S_x} \rightarrow \infty$ => $\hat q_{vS} \rightarrow \hat q_{v_A}$ (no evaporation)

- Diagnostic for synoptic (2m and 10m)-levels D:
  - ○ according turbulent distance δ
  - ○ along vertical CFL-profiles
  - ○ employing z_0 of "SYNOP-lawn: z_0^{dia}

$$\hat\phi_D = \hat\phi_S + \left[\frac{r^\phi_{SD}}{r^\phi_{SA}}\right]_{z_0 = z_0^{dia}} \cdot \left(\hat\phi_A - \hat\phi_S\right)$$



S^{lus}_{ai}= 'c_lnd' + 'plcov' · 'lai'

Specific roughness-treatment over Sea-Surfaces:

- **Sea surface-roughness length is dependent on surface shear:**

dynamic contribution

laminar limit

$$z_0 = \alpha_0 \frac{u_*^2}{g} + \alpha_1 \cdot \frac{k^M}{u_*}$$

Charnock parameter with empirical dependency on 10m-wind-speed

'alpha0'=0,0123

'alpha1'=0,75

surface shear (including impact by SGS wind)

$$u_*^2 = K_0^M \sqrt{F_{T\,0}^M}$$

diffusion coefficient in the surface layer

- **As u_* is also dependent on z_0, the solution is taken through direct time step iteration.**

- **'imode_charpar' > 1 :**
  - **for lakes:**

$$\alpha_0 = 0.1 \cdot \text{'}\mathbf{alpha0}\text{'}$$

  - **else:**

$a = 6.0 \cdot 10^{-3}$ $b = 5.5 \cdot 10^{-4}$ $c = 4.0 \cdot 10^{-5}$ $d = 6.0 \cdot 10^{-5}$

$u2 = 17.5$ $umax = 40.0$ $\underline{\text{'}\mathbf{alpha0\_max}\text{'}} = 0.0335$ $\underline{\text{'}\mathbf{alpha0\_pert}\text{'}} = 0.0$

$ulim = \min\{v_{10m}, umax\}$ $ured = \max\{0.0, ulim - u2\}$

$$\alpha_0 = \min\left\{ \underline{\text{'}\mathbf{alpha0\_max}\text{'}}, \max\left\{\text{'}\mathbf{alpha0}\text{'}, a + \underline{\text{'}\mathbf{alpha0\_pert}\text{'}} + ulim \cdot \left(b + c \cdot ulim - d \cdot ured\right)\right\} \right\}$$

- **'imode_charpar' = 3 :**

$$\alpha_0 = \min\left\{\alpha_0, 0.8 / \max\left\{1.0, v_{10m}\right\}\right\}$$

Main characteristic of TURBTRAN compared to Monin-Obuchov schemes:

- **Direct application** of the **turbulence scheme at two nodes of the vertical axis** within the **NS constant-flux layer**:
  - a) top of the **R**(oughness)-Layer: 0-level
  - b) at the lowest atmospheric boundary-level above: P-level

- **Vertical integration** of the **flux-gradient relation** through the **constant-flux transfer-layer** below level P:
  - In order to derive **bulk transfer resistances**
  - Using consistently chosen **interpolation functions for diffusion coefficients** between the two nodes.
  - Extending the **resistance-calculation to the R**(oughness)-**Layer and the L**(aminar)-**Layer (**adjacent to the earth surface)

⇨

- **Substitution** of an **artificial "long-tailed"** dependency on **bulk-Ri-numbe**r for stable stratification:
  - by the impact of **STIC-terms** introduced **at the upper node (P)**
  - which automatically **avoids excessively increasing Ri-numbers**

- **Substituting** the provision of a **specific roughness-length for scalars**:
  - by a direct **calculation of the related R- and L-Layer resistances**

Questions?

Further development related to TURBDIFF:

▪ Adaptation or substitution of empirical parameterizations (e.g. minim. diff.coeff.)	**continuous action**
▪ Inclusion of cloud-ice in turbulent saturation adjustment – Mixed water-ice phase	**prepared** (but set to lower priority)
▪ Consolidation of the STIC concept related to – TDC (-> thermal SSO effect), SHS and CON – Consideration of all the related non-turbulent SGS vertical transport – Combined saturation-adjustment with contributions from all SGS patterns ○ associated cloud-diagnostics as input for radiation ○ GS thermal impact of SGS-condensation	**not yet in ICON-master** **running** **in preparation** pragmatic approach already present
▪ *Extension with prognostic equations for scalar (co-)variances (TKESV)* – *Including non-turbulent properties (skewed distribution functions, length-scale of coherent convective motions) <-> is in contradiction to STIC!!*	***test code existing***
▪ Introduction of further 3D-extensions: – TKE-advection – Horizontal turbulent diffusion – Diffusion by horizontal shear eddies	**not yet activated in ICON dynamics code for NWP** **horiz. diff.-coeffs. ready**
▪ Introduction of all roughness layer terms (metric SAI-terms from SSO)	**in preparation**
▪ Full Documentation	**running**

Further development related to TURBTRAN\|TERRA:

▪ Activating the hyperbolic u^{ϕ}-interpolation for stable stratification	being tested and consolidated
▪ Introducing missing STIC-impact at 0-level (surface-layer shear-amplification)	being tested and consolidated
▪ Treatment of laminar effects without a laminar layer separation	turbulent/(molecular resistance-integral derived analytically
▪ Considering additional roughness due to tile variation of land-use	test-version in ICON-branch
▪ Revised formulation of 10m wind-speed and -gusts valid within the roughness layer or at exposed grid points on mountain tops	being investigated
▪ Introduction of a soil-covering layer with its own implicitly coupled heat budget in TERRA – Implicit Treatment of major Surface-Processes (ISUP) – Implicitly coupled heat budgets for soil, snow-free surface-cover and (multi- or single-layer) snow cover – Including a revised implicit treatment of interception water	test-version in ICON-branch
– Semi-transparent and substantial land-use layer (canopy)	proto-type to be taken from a COSMO-test-version
▪ Consideration of a vertically resolved part of the roughness layer from land-use	prepared
▪ Full Documentation	running